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Physical explanations of Einstein's gravity

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Abstract

Einstein's gravitational field equations from his general theory of relativity have formed the foundations of gravitational studies since their publication. His work is widely acknowledged as an example of a theoretical study that made a great contribution to our understanding of gravity. Einstein's mathematical approach has made the topic complex and open to misinterpretation. This study evaluates the physics upon which his mathematics operates. It shows that mass distorts space–time by the redshift of photons. It derives two alternative metrics to the Schwarzschild metric. One was derived directly from Einstein's early work on gravity. The other was derived from Einstein's field equations by removing the approximations introduced in the Schwarzschild metric derivation. Both match observation better than the Schwarzschild metric and show why Einstein did not believe in black holes. The metric derived directly from Einstein's early gravitational study, predicts the torus shape shown in the Event Horizon Telescope collaboration image. In showing the physics involved, this study suggests it is easier to understand the complexities of his work.

1. Introduction

In the modern era, Copernicus (1543) suggested that the Sun was the centre of motion. Brahe (1572) made detailed studies of the motion of the planets. His assistant, Kepler (1609, 1619), used Brahe's observations in his work that showed the planets were orbiting the Sun. He determined three laws of planetary motion around the Sun. Their contributions have been summarized [1].

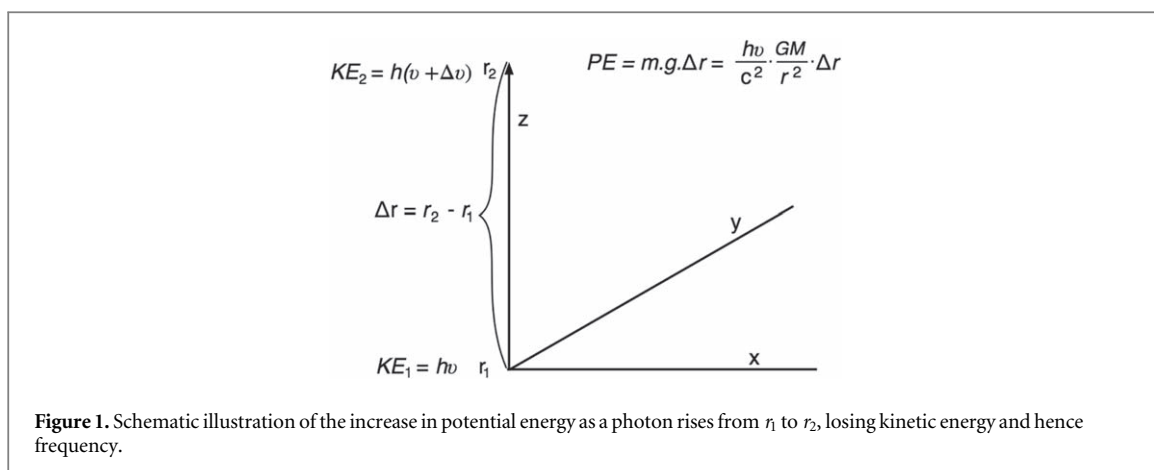
With his construction and use of a telescope, Galileo Galilei [2, 3] was able to observe the motions of the planets with greater accuracy. He also observed that Jupiter had four moons orbiting it. His work confirmed the idea that the planets moved around the Sun and that gravity was responsible for those heavenly motions. Between them all, they firmly established the solar centric model for the motion of the planets around the Sun.

With that as a background, Newton [4, 5] placed gravity and gravitational effects on a firm foundation. He introduced his universal law of gravitational attraction between two bodies of masses M and m , through the equation

$$F_G = \frac{GMm}{r^2} \quad (1)$$

where F_G is the gravitational attraction between the two bodies, G is Newton's universal gravitational constant and r is the distance between their centres of mass. It was suggested that Huygens derived a similar expression a few years earlier, but never published [6]. Newton determined that, when the gravitational force varied as the inverse square law of the distance between their centres of mass, the planet would prescribe an ellipse about the Sun, with the Sun being at the centre of one of its focal points. In his Proposition 45, volume 1, Newton also showed that, if gravity were weaker than inverse square law, a planet's perihelion would precess in its direction of motion. If it were stronger than inverse square, its perihelion would regress against its direction of motion.

In his Proposition 31 Newton introduced his shell theorem. A mass inside a spherical shell of uniform thickness and density would not experience any gravitational attraction from that shell, irrespective of its position within the shell and its density and thickness. He also showed that gravity, which was believed to control the motion of planets around the Sun, was responsible for holding objects on the surface of the planets. Gravity



was universal. He did not determine the value of G , his universal gravitational constant. Nor did he give a reason for gravity to operate at a distance.

Improvements in telescopes allowed more accurate observations. Newton's work on gravity matched most observations for almost 200 years. Towards the end of the 19th century CE, it was noticed that Mercury had an anomalous orbital effect (Le Verrier [7], Newcomb [8]. Astronomers observed that, viewed from Earth, Mercury's total orbital precession was 5,600 arc sec per century (as/c) [9]. Newtonian mechanics predicted 5,557 as/c . Of that, 5026 as/c were due to the precession of Earth's axis of rotation.

The gravitational attraction of the planets gave Mercury's perihelion an additional precession of 532.2 as/c . That left a difference of almost 43 as/c . That was verified by Clemence [10] and slightly refined by Park *et al* [11]. That was the difference explained by Einstein [12–15] in his publications of the gravitational field equations from his theory of general relativity.

Those publications were of great mathematical complexity. Most people found them difficult to follow. They are widely acknowledged as an example of a theoretical study that made a great contribution to our understanding of gravity. Their complexity and apparently pure mathematical derivation led some to question the validity of his work. Supporters of his field equations acknowledge his calculations must be correct because they matched every observation against which they were tested.

His work made significant advances over Newton's work. His prediction that gravity was caused by mass distorting space–time overcame Newton's concern of action at a distance. He also predicted the anomalous precession of Mercury's perihelion. Other predictions included that time would be slower near a massive object and faster away from it. It was associated with radial distances becoming longer near a massive object and shorter away from it. He also predicted the trajectory of light rays would be altered by the presence of mass, a rotating mass would drag space–time with it and that two bodies rotating about their common centre of mass would emit gravity waves.

A prediction that came from solutions to his field equations is the existence of black holes. They have no basis in physics. They are said to be an exact solution to Einstein's gravitational field equations. Einstein did not believe in their existence. Recent observations by the Event Horizon Telescope collaboration [16, 17] appear to confirm their existence. Did that make Einstein wrong?

It is suggested that an appraisal of the physics involved in mass distorting space–time would give the physical reasons for his predictions. Whitehead suggested it was only possible to derive the space–time metrics from mathematical considerations [18]. The objective of this appraisal is to show the physics behind mass distorting space–time. From that it becomes possible to determine which effects are real and why.

2. Background

Einstein [19, 20] provided the background to his work. In his 1911 paper he studied the influence of Newtonian gravity on the propagation of light. It is suggested he was able to do that because of his earlier work. In his study on the photoelectric effect, Einstein [21, 22] determined that electromagnetic radiation was transmitted as discrete packets of energy, E , given by $E = h\nu$, where h is Planck's constant and ν is the radiation's frequency. Those discrete packets are now called photons. They travel at the speed of light, c . Their frequency, ν , gives them wavelength, λ , related through $c = \nu\lambda$.

In that same year Einstein published his special relativity theory in which changes in length, time and mass occurred with increasing velocity [23, 24]. He established the relationship between mass and energy as $E = mc^2$. He also established that photons had mass and imparted inertia between bodies [25, 26]. From that he derived

the expression for photon mass, m_p , namely

$$m_p = \frac{hv}{c^2} \quad (2)$$

Photons having mass and responding to gravity were verified experimentally [27–30]. That m_p was the mass to which Einstein [19, 20] applied Newton's inverse square law of gravity to determine the effects it had on the propagation of light.

Figure 1 shows a schematic illustration of a photon, energy $KE_1 = hv_1$ at r_1 , moving distance $\Delta r = r_2 - r_1$ vertically against gravitational attraction. It was adapted from Einstein [19, 20]. Projectiles are constant mass particles that lose speed as they rise against a gravitational field. Photons are constant speed particles that lose frequency and hence mass as they rise against a gravitational field. Both are constant energy particles that gain potential energy as they lose kinetic energy when moving against a gravitational field of strength

$$g = \frac{GM}{r^2} \quad (3)$$

where r is the distance from the centre of mass of the object.

A photon rising distance Δr from r_1 to r_2 gains potential energy given by

$$PE = \frac{hv}{c^2} \frac{GM}{r^2} \Delta r$$

Conservation of energy means that $KE_2 = KE_1 + \Delta PE$, giving

$$hv_2 = hv_1 + hv_1 \frac{GM \Delta r}{c^2 r^2} \quad (4)$$

Re-arranging equation (4) gives

$h(v + \Delta v) = hv - hv \frac{GM \Delta r}{c^2 r^2}$. Dividing by h and re-arranging gives

$$\frac{\Delta v}{v} = \frac{GM}{c^2} \frac{\Delta r}{r^2} \quad (5)$$

G , M and c are constants that can be combined to give $\alpha = \frac{2GM}{c^2}$. Inserting α into equation (5) and setting the limit $\Delta r \rightarrow 0$ gives

$$\frac{dv}{v} = \frac{\alpha dr}{2r^2} \quad (6)$$

Integrating equation (6) from r_1 to r_2 gives

$\ln v_2 - \ln v_1 = -\frac{\alpha}{2r_2} + \frac{\alpha}{2r_1}$, which simplifies to

$$v_2 = v_1 e^{(\alpha/2r_1 - \alpha/2r_2)} \quad (7)$$

Equation (7) can be applied to the frequency change of photons traveling from the Sun's surface to Earth. The sun's Schwarzschild radius, α , is 2.954 km. Its surface radius is 7×10^5 km. Earth's orbit radius is

1.49×10^8 km from the Sun. Substituting into equation (7) gives $\frac{\Delta v}{v} = e^{\left(\frac{2.954}{1.4 \times 10^6} - \frac{2.954}{2.98 \times 10^8}\right)} = 2.1 \times 10^{-6}$. Within experimental error, that is the same as was reported by Einstein [19, 20] and verified by observation [31–33].

That change in frequency is the reason why observers in a low gravity field see known frequencies from a strong gravity field at a lower frequency. It is the cause of time dilation in a high gravity field. Einstein showed that time dilation caused photons to be bent by a strong gravitational field. His calculation did not include a term for space distortion. His prediction for bending of light rays passing close to the Sun's surface was only half the answer.

Equation (7) was as far as Einstein took those calculations. In his calculations, he used the linear form, $\alpha/2r$. The above presented the work in greater detail, using the exponential form $e^{\alpha/2r}$. For very low deflections, the results are indistinguishable.

3. Continuation

Through $c = v\lambda$, a change in frequency automatically generates a change in wavelength. When a photon's wavelength is changed, it is called a redshift and denoted z . It can be due to a Doppler effect or gravity and is given by

$$z = \frac{\lambda_r - \lambda_0}{\lambda_0}$$

where λ_0 is its original wavelength and λ_r is its wavelength at distance r from its origin.

Using $c = v\lambda$, equation (7) can be re-written to give

$$\lambda_2 = \lambda_1 e^{(\alpha/2r_2 - \alpha/2r_1)}$$

Setting $\lambda_1 = \lambda_0$ and $\lambda_2 = \lambda_r$ gives

$$z = e^{\alpha/2r} - 1 \quad (8)$$

When that reduces to

$$r \gg \alpha, \quad (8a)$$

$$z = \alpha/2r \quad (8a)$$

Einstein [19, 20] showed that time distortion was given by the change in photon frequency caused by the strong gravitational field. In the same manner, space distortion is caused by the change in photon wavelength. A Newtonian distance d_N becomes the relativistic distance, d_{Rel} , from the relationship

$$d_{Rel} = d_N(1 + z) = d_N e^{\alpha/2r} \quad (9)$$

Newtonian time, t_N , becomes the relativistic time, t_{Rel} , related by

$$t_{Rel} = \frac{t_N}{(1 + z)} = t_N e^{-\alpha/2r} \quad (10)$$

Newtonian distances and times are replaced by Einstein's relativistic distances and times.

Equations (9) and (10) are only good for determining what happens to bodies moving perpendicularly towards or away from the centre of mass of a massive object. They are no good for determining what happens in non-radial travel. For that, it helps to calculate the space-time metric.

Minkowski [34, 35] merged space and time into a four-dimensional space-time continuum, given by:-

$$s^2 = c^2 t^2 - x^2 - y^2 - z^2$$

Where s is the space-time co-ordinate, x , y and z are the orthogonal Cartesian co-ordinates and t is time.

Obtaining the space-time metric involved converting Minkowski's equation into a differential form and transferring from Cartesian to polar co-ordinates. It has been done many times and is virtually a standard mathematical look up formalism. To help understand it, the following is a summary of the procedure used by Schwarzschild [36]. It also includes the additions for the distortions of space and time by a gravitational field.

The Cartesian differential of Minkowski's equation becomes

$$ds^2 = Fdt^2 - H\{dx^2 + dy^2 + dz^2\} - J\{xdx + ydy + zdz\},$$

where F , H and J are functions of r when $r = \sqrt{x^2 + y^2 + z^2}$. They take space-time distortion into consideration. Changing these Cartesian co-ordinates to Polar co-ordinates gives $ds^2 = Fdt^2 - (H + Jr^2)dr^2 - Jr^2(d\theta^2 + \sin^2 \theta d\varphi^2)$. To that we must now add the space and time distortion factors produced by the mass. From equations (9) and (10) we get

$F = e^{-\alpha/r}$, $H + Jr^2 = e^{\alpha/r}$ and $J = 1$. Inserting those gives

$$ds^2 = dt^2 e^{-\alpha/r} - dr^2 e^{\alpha/r} - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (11)$$

A Maclaurin/Taylor series expansion of $e^{-\alpha/r}$ yields

$$e^{-\alpha/r} = 1 - \alpha/r + \alpha^2/r^2 2! - \alpha^3/r^3 3! \dots + ((-\alpha)^n / r^n n!)$$

For $r \gg \alpha$, a first approximation of equation (11) becomes

$$ds^2 = dt^2 \left(1 - \frac{\alpha}{r}\right) - \frac{dr^2}{\left(1 - \frac{\alpha}{r}\right)} - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (12)$$

Equation (12) is one of the accepted solutions to Einstein's field equations [37]. Other accepted alternative equations include c^2 associated with the dt^2 term and a reversal of polarity. Exact solutions from Minkowski's space-time continuum should include c^2 . Normalization of constants to 1 is often used in mathematical physics. The reversal of signs is associated with calculations from different directions.

Equation (11) is referred to as a 'weak field' solution to Einstein's gravity. Equation (12) and its variants are referred to as the 'strong field' solutions to Einstein's field equations. They predict that gravity is stronger than Newton's inverse square law.

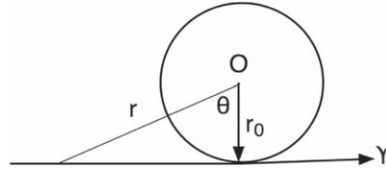


Figure 2. Schematic illustration of the deflection of a photon passing at distance r_0 from the centre of mass of a massive object at O.

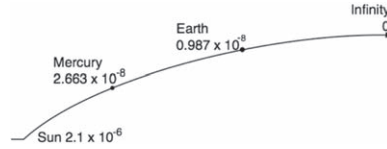


Figure 3. Schematic illustration of the Sun's redshift at different positions from its centre of mass.

Table 1. The physical values needed to calculate Mercury's anomalous orbital precession.

Physical property	Value
Speed of light c	$2.9979 \times 10^5 \text{ km s}^{-1}$
Newton's universal gravitational constant G	$6.6743 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$
Sun's mass M	$1.989 \times 10^{30} \text{ kg}$
These values of c , G and M give α	2.954 km
Mercury's semi major axis r_0	$57.91 \times 10^6 \text{ km}$
Mercury's orbital eccentricity e	0.2056
These values give redshift at Mercury orbit z_M	2.663×10^{-8}
Earth orbital radius	$149.6 \times 10^6 \text{ km}$
These values give redshift at Earth orbit z_E	0.987×10^{-8}
Mercury orbital period M_{Orb}	88 Earth days
Earth orbital period E_{Orb}	$365.25 \text{ Earth days}$

4. Matching some observations

Equation (7) already showed the calculations matched the observed redshift of photons from the Sun. Other predictions were gravitational lensing and the Mercury's anomalous orbital precession.

4.1. Gravitational lensing

Figure 2 shows the passage of a photon, γ , travelling past a massive object of Schwarzschild radius α , making its closest approach at distance r_0 from the centre of mass. At distance r from the centre of mass, O, it will experience an instantaneous space distortion $d\Delta_{dist} = \alpha/2r$. The total space distortion it experiences is given

$$\Delta_{dist} = \int_{-\infty}^{+\infty} \alpha/2r dr. \text{ Since } r = r_0/\cos \theta, \text{ that becomes } \Delta_{dist} = \int_{-\pi/2}^{+\pi/2} \frac{\alpha \cdot \cos \theta d\theta}{2r_0} = \frac{\alpha}{r_0}.$$

Equations (9) and (10) show that when there is a change in space distortion, there is a corresponding change in time distortion. For small distortions, i.e., $\gg \alpha$, that distortion is of equal magnitude and opposite sign.

Equation (11) shows that

the total space–time distortion, Δ_s , is obtained by subtracting the two distortions. That gives

$$\Delta_s = \frac{2\alpha}{r_0} \quad (13)$$

For the Sun, $\alpha = 2.954 \text{ km}$ and $r_0 = 700,000 \text{ km}$. Inserting those into equation (13) gives $\Delta_s = 4.22 \times 10^{-6}$ radian, or 1.74 arc seconds. That is the expected distortion of photons passing close to the Sun's surface. Photons measured at Earth have not travelled back to infinity. That slight decrease indicates the measured deflection will be closer to 1.73 arc seconds. Those calculations agree with Einstein's prediction within the approximations he used and have been verified by observation [38, 39].

It should be noted that $\alpha/2r_0$ is the redshift of photons from their distance of closest approach to the massive object. The mathematics of gravitational lensing is that the total deflection, in radians, of a photon passing close to a massive object is four times the redshift of photons at their position of closest approach to the centre of mass.

4.2. Anomalous precession of Mercury's perihelion

Observed from Earth, Mercury's total orbital precession is 5,600 *as/c*. Newtonian mechanics explained all but about 43 *as/c*. Einstein [12] explained that difference through his equation, $\varepsilon = 24\pi^3 \frac{a^2}{T^2 c^2 (1 - e^2)}$. More accurate measurements now give his prediction of Mercury's anomalous precession as 42.98 *as/c*.

Under this interpretation of Einstein's work, equation (9) indicates that Newtonian distances become relativistic distances when multiplied by $(1 + z)$, where z is the gravitational redshift produced by the Sun at the distance of interest. Figure 3 schematically illustrates the Sun's redshift at different distances from its centre of mass. Those figures were calculated from equation (8), using the data given in table 1.

Mercury's corrected orbit radius $r_M = r_0(1 - e^2)$, becomes $r_M(1 + z_M)$. That increase in radius can only occur if the Sun's gravitational attraction is less than inverse square. It means that Mercury has to travel a greater distance to complete an orbit from perihelion to perihelion. That increased circumference, given by $\Delta_{Circ} = 2\pi r_M(1 + z_M) - 2\pi r_M = 2\pi r_M z_M$, is the extra distance Mercury travels to complete an orbit. Its angular precession per orbit, ε_M , is given by $r_M \cdot \varepsilon_M = \Delta_{Circ}$ or $\varepsilon_M = 2\pi z_M$.

Figure 3 shows that Earth's orbit is also affected by the Sun's gravitational attraction. The space distortion between Mercury and Earth must be corrected for the Sun's redshift at Earth. That gives

$$\varepsilon_M = 2\pi(z_M - z_E) \quad (14)$$

Mercury's orbital period is 88 Earth days. In 100 Earth years, it will make 415 orbits around the Sun. During that time, Earth makes 100 orbits around the Sun. From Earth, Mercury will only be seen to make 315 orbits. As observed from Earth, equation (14) becomes

$$\varepsilon_M = 2\pi(z_M - z_E) \left(1 - \frac{M_{Orb}}{E_{Orb}}\right) \text{radian/orbit} \quad (15)$$

Inserting the appropriate values from table 1, converting from radians to arc seconds and multiplying by the number of Mercury orbits per Earth century gives $\varepsilon_M = 42.99$ *as/c*.

As mentioned earlier, the most up to date figures applied to Einstein's calculations gave 42.98 *as/c*. There are suggestions that the value observed by Park *et al* [11] differs from the predicted value by ≈ 1.2 *as/c*, a value considerably larger than the 0.01 difference between this approach and Einstein's approach. That indicates the two approaches are the same. It also indicates this approach explains the physics that underpins Einstein calculations. Namely that mass distorts distances, increasing lengths by the redshift z of photons. It is suggested this calculation is easier to follow than derivations by Einstein and others. It has the advantage it can be extended to other situations.

Einstein's theories are called relativity because all measurement results depend upon the position of the observer. There is no absolute reference point or frame. His theories enable observers in different places to determine the results obtained by other observers in different reference frames when the speed of light is constant for all observers.

Equation (15) can be modified to obtain the general equation for Mercury's anomalous precession per orbit for observers at different positions within the solar system. It becomes

$$\varepsilon_M = 2\pi(z_M - z_O) \left(1 - \frac{M_{Orb}}{O_{Orb}}\right) \text{radian/orbit} \quad (16)$$

where z_0 is the Sun's redshift at the observer's position and O_{Orb} is the orbital period of the observer around the Sun. Table 2 gives the expected results for some different positions associated with the solar system. Those results are additional to the 532.2 *as/c* due to the precession caused by the gravitational pull of the other planets on Mercury.

Notable features of table 2 are that an observer from beyond the solar system would see Mercury's anomalous orbit precess an additional 90 arc seconds when they had seen Earth make 100 complete revolutions around the Sun. Observers accelerated in retrograde Earth orbit and held in place by a large solar sail, would see Mercury's anomalous orbit precess by 56.63 arc seconds per Earth century. The further away is the observer from the Sun, the larger will be the observed anomalous precession. Table 2 demonstrates that observations are relative to the position of the observer.

In the same manner, the general equation for the precession of a satellite orbiting the Sun and measured from Earth, in radians per satellite orbit, becomes

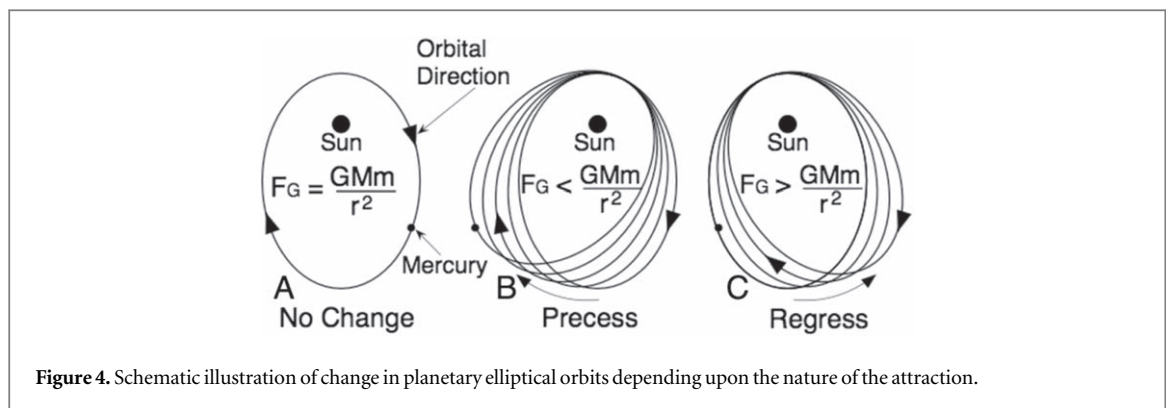


Table 2. The precession of Mercury's orbit in arc sec per Earth century as measured from different positions.

Observer position	$z_O \times 10^{-8}$	M_{Orb}/O_{Orb}	Per Earth century, arc sec
Flat Minkowski space-time	0	∞	89.99
Stationary at Earth Orbit	0.987	∞	56.63
Earth Orbit	0.987	0.241	42.99
Mars Orbit	0.648	0.128	59.38
Saturn Orbit	0.103	0.0082	85.86

$$\varepsilon = 2\pi(z_S - z_E)\left(1 - \frac{S_{Orb}}{E_{Orb}}\right) \quad (16a)$$

where z_S and S_{Orb} are the Sun's redshift at the satellite's corrected orbit and its orbital period respectively. The closer the satellite orbited the Sun, the greater are other effects such as the Sun's oblateness and its radiation and solar wind.

The general expression for the precession, in radians per orbit, of any object about a mass M external to the solar system is given by

$$\varepsilon = 2\pi\left(\frac{GM}{c^2(1 - e^2)r_O}\right) = 2\pi\frac{\alpha_M}{2r_{OC}} = 2\pi z_{OC} \quad (17)$$

where r_O is the orbiting body's semi major axis, α_M is the massive body's Schwarzschild radius and z_{OC} is its redshift at the orbiting body's corrected semi major axis r_{OC} .

Newton predicted that, if gravity was inverse square a planet would prescribe an ellipse around the Sun, with its aphelion and perihelion always returning to the same points, as shown in figure 4(A). If gravity were weaker than inverse square, the planet travels further away from the Sun and under the influence of a weaker force. That greater distance causes it to return to its aphelion and perihelion positions a little later. It will precess in its direction of orbit, as shown in 4B.

If gravity were stronger than inverse square, the planet would orbit a little closer to the Sun. It would have to travel a shorter distance under the influence of a stronger force to return to its aphelion and perihelion positions. They would be reached a little earlier. Its perihelion would regress in the opposite direction to the orbiting planet, see figure 4(C).

It should be noted that the accepted Schwarzschild metrics, equation (12) and its variants, all have the term $1 - \frac{\alpha}{r}$. They are recognized as the strong field solution to Einstein's gravitational field equations. They all predict that gravity will be stronger than inverse square. By the mechanism worked out by Newton and presented above, it is not possible for an orbit's perihelion to precess in its direction of travel if gravity is stronger than inverse square.

That poses a problem for those who accept the Schwarzschild metric as an exact solution to Einstein's gravitational field equations.

It is herein suggested the mathematical complexity associated with Einstein's calculations made it easy to overlook the sign and interpret a predicted regression as a predicted precession. As Newton showed, and supported by this presentation, there is simply no physical way in which a gravitational field that is stronger than inverse square can cause an orbiting body to precess in its direction of orbit.

Orbital precession requires gravity to be weaker than inverse square so that a body has to travel further to complete its orbit from perihelion to perihelion. Einstein did not calculate Mercury's total orbit and derive the

difference between his and Newton's calculations as 43 *as/c*. He calculated only the difference his theory introduced to Newton's calculations.

The approach used in this derivation, based on photon redshift, was similar. It started with Newton's gravity and added its effect on photons. It showed the new orbital distance that Mercury had to travel to complete a perihelion to perihelion orbit was $2\pi r_M(1 + z_M)$. Mercury's Newtonian orbit of $2\pi r_M$ was subtracted to arrive at its anomalous precession of $2\pi z_M = 42.99 \text{ as/c}$. Both approaches have a close relationship with Newton's calculations.

Even if Mercury's orbit were circular, that difference would have been picked up because changes in its orbit were first determined by its transits across the Sun between 1677 and 1881 [8]. The elliptical orbit calculation makes the effect easier to recognize precession in other orbits. A change in orbital radius of a distant object would not be recognised.

5. Comments on Einstein's calculations

The majority of Einstein's 1916 paper on 'The Foundations of the General theory of Relativity,' [14] was taken up with tensor studies, the purpose of which included establishing the relationship between the four-dimensional space-time co-ordinates under the influence of gravity. Whether tensors clarify or confuse the reader, depends upon the reader's understanding of them.

The above redshift calculations have matched Einstein's predictions of photon redshift, gravitational lensing and orbital precessions. They indicate the process by which mass distorts space-time is through changes in the wavelength and frequency of photons. It becomes a philosophical question as to whether mass distorts space-time to produce gravity that distorts photons, or gravity is produced by mass distorting photon wavelengths, which equates to space-time distortion. In either event, the calculations are the same.

The redshift metric of equation (11) and its associated calculations do not match predictions under the Schwarzschild metrics of equation (12) and its equivalents. The latter predict that gravity is stronger than expected under Newton's inverse square law. At the same time the calculations for equation (15) show that, in order for Mercury's orbit to precess, gravity must be weaker than inverse square. That was predicted by Newton. Did Einstein make a mistake in his calculations?

With many people not understanding the complexity of his work, some suggest he did. In his publication on The Foundation of the General Theory of Relativity, Einstein introduced approximations. After deriving his field equations, (47), he pointed out '*There is only a minimum of arbitrariness in the choice of these equations*'. He went on to point out the reasons the equations were not exact. In that passage he acknowledged his use of approximations.

In section 22 of his Foundations paper, Einstein [14, 15] derived $g_{11} = -\left(1 + \frac{\alpha}{r}\right)$. From that it follows that, for $r \gg \alpha$, $dx = \frac{1}{\left(1 + \frac{\alpha}{2r}\right)}$. He went on to indicate that '*.. correct to a first order of small quantities* $dx = 1 - \frac{\alpha}{2r}$..

(71)'. When $\frac{\alpha}{2r} \approx 2 \times 10^{-8}$, see figure 3, his equation (71) is a very valid approximation. Equation (8) show $\frac{\alpha}{2r} = z$.

After his equation (71), he stated '*The unit measuring rod thus appears a little shortened in relation to the system of co-ordinates by the presence of the gravitational field, if the rod is laid along a radius*'. That could appear to be in conflict with equation (9), which points out that relativistic lengths were longer than Newtonian lengths by $(1 + z)$. In its full context, it '*appears a little shortened*', by $(1 - z)$, because the background against which it was observed was lengthened by $(1 + z)$.

There is nothing in and around Einstein's equation (71) that could be construed as gravity was stronger than inverse square, thus shortening lengths.

Additionally, the above work is in agreement with his calculation that lengths increase by $(1 + z)$. Both approaches find that, '*correct to a first order of small quantities*', mass increases length by the redshift z . That can only occur if gravity is weaker than inverse square, a pre-requisite condition for an orbiting body's perihelion to precess.

Apart from his approximations, it could be stated there is one other problem associated with his work. It appears so complex that most of those who tried to follow it were confused. That is not an error by Einstein. His approximations appear to have caused problems for those who attempted to provide exact solutions to his field equations.

Two examples of field equations, Einstein's original, his 47, and a modern Ricci tensor format, equation (22), are given below.

$$\frac{\partial \Gamma_{\mu\nu}^{\alpha}}{\partial x_{\alpha}} + \Gamma_{\mu\beta}^{\alpha} \Gamma_{\nu\alpha}^{\beta} = 0 \quad (\text{E47})$$

$$\sqrt{-g} = 1$$

$$R_{\mu\nu} - 0.5Rg_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (22a)$$

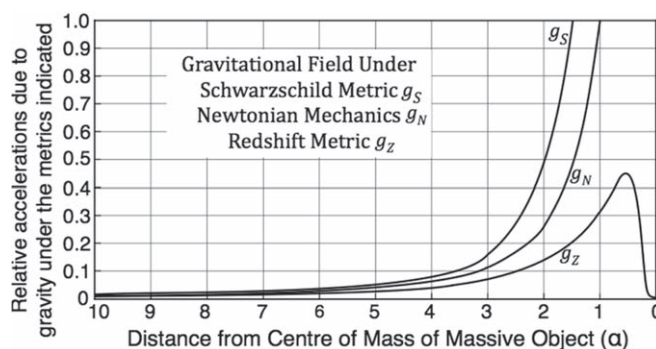


Figure 5. Plots of the relative accelerations due to gravity under the different equations given in table 3.

Table 3. The gravitational attraction and accelerations due to gravity for different distances from the centres of mass of objects of any mass M .

Gravity theory	Acceleration due to gravity
Redshift	$g_Z = \frac{GM}{[r(1+z)]^2} = \frac{GM}{r^2 e^{\alpha/r}}$
Newton	$g_N = \frac{GM}{r^2}$
Schwarzschild	$g_S = \frac{GM}{[r(1-z)]^2} \approx \frac{GM}{\left(1 - \frac{\alpha}{r}\right)r^2}$

Their generally accepted solutions take one of the variations in equation (12). The problem posed by those solutions is that they are exact solutions to the approximations mentioned above. Exact solutions to approximations are still approximations.

6. Comparing the different gravitational predictions

The above has shown that Mercury's anomalous orbital precession was caused by gravity being weaker than inverse square law. That extends the circumferential distance Mercury had to travel to return to its perihelion position in each orbit. That additional distance was given by the redshift z . An object of mass m at a distance r from the centre of mass of a body of mass M experiences a gravitational attraction $F_G = \frac{GMm}{r^2}$ and travels a distance $2\pi r$ to make one complete revolution. For the circumference to increase from $2\pi r$ to $2\pi r(1+z)$, the gravitational attraction must decrease to

$$F_G = \frac{GMm}{[r(1+z)]^2} \quad (18)$$

Inserting equations (8) into (18) gives

$$F_G = \frac{GMm}{r^2 e^{\alpha/r}} \quad (19)$$

from which the acceleration due to gravity becomes

$$g_Z = \frac{GM}{r^2 e^{\alpha/r}} \quad (20)$$

The gravitational field equations derived by Einstein, plus the variants by others following his work, are general equations from which the gravitational acceleration due to the presence of the mass M can be calculated. Equation (20) gives the direct expression for the acceleration due to gravity at different distances, r , under this redshift study. In the same manner, equation (3) gives the acceleration due to gravity for any distance, r , under Newtonian gravity.

Table 4. The accelerations due to gravity for different distances under the different theories, with distances expressed as functions of the Schwarzschild radius α .

Distance	Redshift	Newton	Schwarz-schild
r/α	$g_Z \propto 1/r^2 e^{\alpha/r}$	$g_N \propto 1/r^2$	$g_S \propto 1/\left(1 - \frac{\alpha}{r}\right)r^2$
10	9.048×10^{-3}	1.00×10^{-2}	1.11×10^{-2}
9	1.101×10^{-2}	1.23×10^{-2}	1.263×10^{-2}
8	1.377×10^{-2}	1.56×10^{-2}	1.783×10^{-2}
7	1.899×10^{-2}	2.04×10^{-2}	2.380×10^{-2}
6	2.353×10^{-2}	2.78×10^{-2}	3.366×10^{-2}
5	3.618×10^{-2}	4.00×10^{-2}	5.000×10^{-2}
4	4.867×10^{-2}	6.25×10^{-2}	8.333×10^{-2}
3	7.963×10^{-2}	0.111	0.167
2	0.1516	0.250	0.500
1.5	0.2282	0.444	1.337
1.0	0.3679	1.000	
0.9	0.4062	1.234	
0.8	0.4475	1.562	
0.7	0.4890	2.041	
0.6	0.5245	2.778	
0.5	0.5413	4.000	
0.4	0.5130	6.250	
0.3	0.3956	11.11	
0.2	0.1648	25.0	
0.1	4.54×10^{-3}	100	
0.05	8.35×10^{-7}	400	

Equations (3) and (20) enable the direct calculation of the gravitational field strength at any distance r from the centre of mass of any object of any mass M . Using the approximations introduced by Einstein, the gravitational force and acceleration due to gravity can be determined for the Schwarzschild metric by replacing r by $r(1 - z)$. Einstein's approximations mean it is only accurate for $r \gg \alpha$. Even then, only if no other errors are introduced.

The equations for the three different theories are shown in table 3. The terms for the acceleration due to gravity can be used to calculate the strength of gravity at any radial distance r from the centre of mass. Table 3 has the advantage over gravitational field equations in that the values can be calculated for any value of M and r . Examples of some calculations for M and r expressed in terms of α , the Schwarzschild radii, are given in table 4.

Table 4's first column gives the various distances expressed as a function of r/α . The other columns give the results for the relative gravitational strengths under the different theories indicated above them. In all cases, the stated results are for bodies moving directly towards the centre of mass of M . The actual accelerations are determined by multiplying the calculated values by GM .

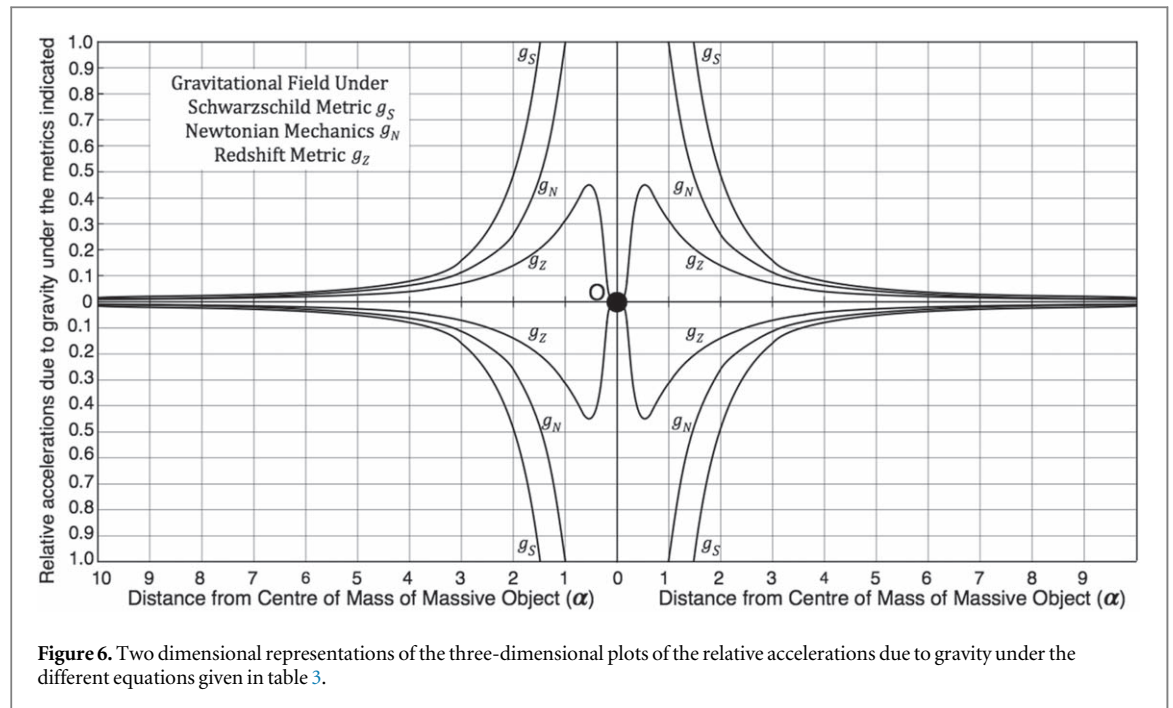
The table 4 results are plotted in figure 5 and joined by smooth curves. As expected from tables 3 and 4, the redshift results, g_Z , are always weaker than the inverse square law. The Schwarzschild results are always stronger than the inverse square law. As the distance from the centre of mass increases, the gravitational strengths tend towards the same value. At $r = 10\alpha$, the differences between the redshift and Schwarzschild results are about 10% either side of the Newtonian acceleration.

At $r \approx 5\alpha$, the three metric accelerations start to differ significantly. Newtonian acceleration tends to infinity at $r = 0$. Gravitational acceleration under the Schwarzschild metric tends to infinity at $r = \alpha$. Both are as expected.

Gravitational acceleration under this redshift metric gets stronger closer to the origin, although it is weaker than inverse square and Schwarzschild acceleration. At $r < 0.5\alpha$, space-time distortion, given by $e^{\alpha/r}$, dominates the inverse square law and gravity becomes weaker as r tends further towards 0.

That may seem like a surprising result. It should be considered from the perspective of Newton's shell theorem. A body anywhere inside a spherical shell of uniform density and any thickness will not experience any gravitational attraction from the shell surrounding it. When applied to a large body like the Sun, it means that there will be no gravitational attraction at the Sun's centre. The pressure will be high, but there will be no gravitational attraction. A similar principle applies to a much more massive object, although for a different reason.

The above offers a simple way of determining the acceleration due to gravity under any of the three theories. It requires knowledge of the mass M of the attracting body, to calculate the Schwarzschild radius α . Convert the



distance, r , from the centre of mass to the position where it is desired to determine the acceleration, to the ratio α/r . Put those values into the appropriate equation from table 3 and calculate the acceleration directly.

Figure 5 displays the strengths of the gravitational fields in one dimension. To appreciate the field in the three spatial dimensions, it is necessary to rotate the curve through 360° about the horizontal axis and through 180° around the vertical axis.

Figure 6 show a two-dimensional slice through a three-dimensional representation of the accelerations due to gravity under the different theories. It was obtained by rotating the curves in figure 5 through 180° about the horizontal axis. That was followed by rotating the compound curves about the vertical axis. They show the accelerations due to gravity, vertical axis, against distance from the centre of mass shown in the horizontal axis. The vertical axis represents the distortions in space–time caused by the mass at O.

What are called the distortions of space–time are the accelerations due to gravity at that point in space. Any particle that experiences a distortion in space–time will be accelerated by that distortion in the direction of increasing distortion.

It is suggested that the g_s equation and curves provide most people with a better understanding of the nature of the Schwarzschild metric than do field equations such as these in equations (21) and (22).

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (21)$$

$$R_{\mu\nu} - 0.5Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (22)$$

As mentioned earlier, the approximations Einstein used in developing his field equations mean that, in the absence of other errors, the Schwarzschild metric equations are only valid for $r \gg \alpha$. They are the conditions under which $1 - \frac{\alpha}{r}$ is observationally indistinguishable from $\frac{1}{(1 + \frac{\alpha}{r})}$.

As is shown in section 7, other errors were introduced in the accepted solutions to Einstein's field equations. They result in the Schwarzschild metric calculations being mathematically incorrect and physically impossible. The failure of Newton's inverse square law to make accurate predictions leaves the redshift metric as the best representation of gravitational effects for space outside matter.

Table 4 and figures 5 and 6 indicate that the gravitational field strengths of the redshift metric are not much different from Newtonian mechanics for $r > 5\alpha$. As such, reasonably accurate predictions of redshift gravitational values can be made using Newtonian mechanics, with a slight end correction for the redshift metric equation.

To understand the effects of the redshift gravity, consider the situation of a ball rolling along the redshift gravity profile at the bottom of figure 6. It will be subject to stronger gravitational attraction as it gets close to $r = 0.5\alpha$. It will reach maximum speed at that distance. From there it will continue on, rising against the

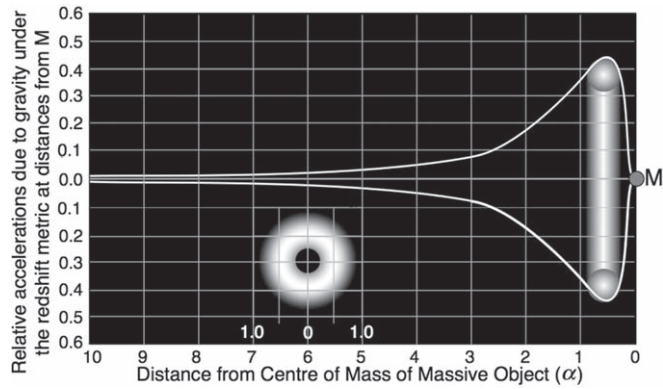


Figure 7. A two dimensional representations of the relative accelerations due to gravity under the redshift metric. The insert shows the expected torus distribution of radiation emitted from an accretion disk that would accumulate around $r \approx 0.5\alpha$.

gravitational attraction. If it is captured by O, it will stay put. Otherwise it will roll back with increasing speed. The same applies to particles under the influence of this redshift metric.

The approximate speed an incident particle will reach as it approaches $r = \alpha$ can be determined by the reverse of the escape velocity. The Newtonian equation for the escape velocity, V_{eN} , for distance r from a gravitationally attracting mass, M , is given by

$$V_{eN} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{\alpha c^2}{r}} \quad (23)$$

Adjusting for the redshift metric gives

$$V_{ez} \approx \sqrt{\frac{\alpha c^2}{r e^{\alpha/2r}}} \quad (23a)$$

Setting $r = \alpha$, equation (23) give the escape velocity for a particle at the Schwarzschild radius of $\approx 0.75 c$. That also means that particles falling inward under the influence of gravity could achieve a velocity up to $\approx 0.75 c$. That shows relativistic velocities can be achieved if incoming particles do not collide with other particles before reaching $r = \alpha$. Electrons reaching speeds $\approx 0.75 c$ have to be accelerated through ≈ 1 MeV. That corresponds to temperatures in the vicinity of $\approx 10^{10}$ K. Those figures depend upon the object being sufficiently massive that its Schwarzschild radius is somewhat larger than double its physical radius.

Particles do not travel along a radius as they approach the centre. Their different trajectories will cause them to collide with other particles close to the centre. With those collisions and other competing factors, it is difficult to determine a definite value for the speed of particles that reach $r = 0.5\alpha$. The best that can be indicated is that they will be relativistic. Collisions will result in significant energy release. These could correspond to temperature up to $\approx 10^{10}$ K. Without further corrections for relativistic effects and the distribution of mass at a radius of 0.5α , that energy release will be concentrated around the radius $r = 0.5\alpha$.

The maximum gravitational strength at $r = 0.5\alpha$ is not a solid surface. Particles moving through it from outside would be attracted back to it if they were not captured by the central mass. Particles moving away from the centre would also be attracted back towards $r = 0.5\alpha$. That would result in the accumulation of high energy particles in the vicinity of $r = 0.5\alpha$. They would move at relativistic velocities in random directions. Rapidly moving particle could move outside that radius. From $g_z = \frac{GM}{r^2 e^{\alpha/r}}$, we can replace GM by $0.5\alpha c^2$ and r by 0.5α to get $g_z = \frac{2c^2}{\alpha e^2}$. That attraction is sufficiently strong that particles with relativistic speeds would be reduced to classical speeds at $r > \alpha$. In the absence of other effects, it appears unlikely that high energy collisions would generate a significant signal at $r > \alpha$.

Figure 7, a reproduction of g_z taken from figure 6, shows the expected result would be the emission of high energy radiation in a torus shape if the incoming particles were preferentially aligned. The insert shows the expected extent of the emissions from within the torus, when viewed perpendicularly to its preferential plane. Its predicted central radius is at $r \approx 0.5\alpha$. The intensities are entirely due to the emissions. They do not require any light bending around the massive object. A feature of the escape velocity equations and figure 7 is that particles can attain high energies, equivalent to temperatures up to $\approx 10^{10}$ K, without the need for the strong gravity predicted by the Schwarzschild solution.

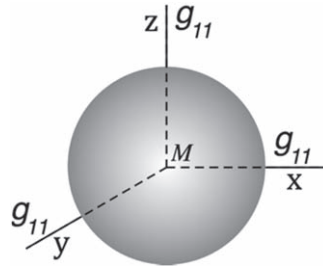


Figure 8. Schematic illustration of how gravity in the x direction has an equal value in the orthogonal y and z directions.

7. The exact solution to Einstein's gravitational field equations

There are several formats to Einstein's gravitational field equations. They all provide solutions that contain the term $\left(1 - \frac{\alpha}{r}\right)$. Equation (12) was derived as a first approximation to an exact solution to the redshift of photons caused by the presence of a gravitational field. Other derivations of equation (12) and its variants are complex. The first solution to Einstein's field equations was provided by Schwarzschild [36]. He introduced an approximation as he derived

$$ds^2 = -dt^2 \left(1 - \frac{\alpha}{R}\right) + \frac{dr^2}{\left(1 - \frac{\alpha}{R}\right)} + R^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$R^3 = \sqrt[3]{r^3 + \alpha^3} \quad (24)$$

When $r \gg \alpha$, equation (24) merges into equation (12). Equation (24) did not predict a singularity at $r > 0$. It did not predict black holes, in which Einstein never believed. It was still an approximation and was rejected in favour of 'exact' solutions.

The equation below is Einstein's equation (53).

$$\frac{\partial}{\partial x_a} \Gamma_{\mu\nu}^\alpha + \Gamma_{\mu\beta}^\alpha \Gamma_{\mu\alpha}^\beta = -K(T_{\mu\nu} - 0.5g_{\mu\nu}) \quad (E53)$$

Like equations (21) and (22), it contains the terms $g_{\mu\nu}$. All 'exact' solutions by different authors have the term $\left(1 - \frac{\alpha}{r}\right)$. The mode of derivation of the solution from the various equations is not as important as the origins of the term $\left(1 - \frac{\alpha}{r}\right)$. That can be determined by evaluating the term $g_{\mu\nu}$. It is common in gravitational field equations and is shown in equation (25).

$$g_{\mu\nu} = \begin{matrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{xx} & g_{xy} & g_{xz} & g_{xt} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{yx} & g_{yy} & g_{yz} & g_{yt} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{zx} & g_{zy} & g_{zz} & g_{zt} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{tx} & g_{ty} & g_{tz} & g_{tt} \end{matrix} \quad (25)$$

The numbers 1, 2 and 3 in $g_{\mu\nu}$ represents the orthogonal directions x, y and z respectively. The number 4 represents time t. The symbol g represents aspects of the gravitational field. For example, g_{11} represents its rate of change in the x direction with x. g_{12} represents the rate of change in the x direction with y. g_{14} represents its rate of change in the x direction with time t. And so forth.

Just before his equation (71), Einstein derived $g_{11} = -\left(1 + \frac{\alpha}{r}\right)$. In radial travel towards or away from the centre of mass, g_{11} can be used as the measure of the change of the g factor towards or away from the centre of mass. Because that change is due to the change in the photon's wavelength, λ , it follows that g_{44} , which is due to the change in frequency, must be the negative inverse of g_{11} . That makes

$$g_{44} = \frac{1}{\left(1 + \frac{\alpha}{r}\right)} \quad (26)$$

Einstein noted that $-1 = g_{11} dx_1^2$. Setting $dx_1 = dx$, because x is the only direction being considered, it follows that $dx = \sqrt{\frac{1}{1 + \frac{\alpha}{r}}}$. As an approximation, when $r \gg \alpha$, that gives $dx = 1 - \frac{\alpha}{2r}$, which is what Einstein stated in his equation (71). In his equation (70), Einstein stated, '.. to the first approximation.. $g_{44} = 1 - \frac{\alpha}{r}$ '. Again, because of equation (26), that is a good first approximation when $r \gg \alpha$.

Table 5. The terms g_{11} as derived by Einstein, as he approximated and as used in the Schwarzschild metrics.

Term	Derived by Einstein	Einstein's approximation	Used in Schwarzschild solutions
g_{11}	$-\left(1 + \frac{\alpha}{r}\right)$	$-\frac{1}{\left(1 - \frac{\alpha}{2r}\right)^2} \approx -\frac{1}{\left(1 - \frac{\alpha}{r}\right)}$	$-\frac{1}{\left(1 - \frac{\alpha}{r}\right)}$
g_{44}	$\frac{1}{\left(1 + \frac{\alpha}{r}\right)}$	$\left(1 - \frac{\alpha}{r}\right)$	$\left(1 - \frac{\alpha}{r}\right)$

Table 5 lists the terms as derived by Einstein, as he approximated them and as others incorporated them into the Schwarzschild metrics. Einstein's derivations involving the term $\left(1 + \frac{\alpha}{r}\right)$ were exact within the approximations he used. It appears his approximations involving $\left(1 - \frac{\alpha}{r}\right)$ were incorporated into what are now called the Schwarzschild metrics. It is believed that led to the 'strong field solutions' of Einstein's field equations.

When $r \gg \alpha$, it makes no difference for the corrections needed for accurate global positioning systems and to be weaker than inverse square law. All forms of the Schwarzschild metric derived solutions use the term $\left(1 - \frac{\alpha}{r}\right)$. As shown in tables 3 and 4 and figures 4 and 5, they can only produce a gravitational attraction that is stronger than inverse square law. They cannot give the observed orbital precession.

In the above calculations, g_{11} was referenced to the x direction. In a radially symmetric situation, there is nothing special about the direction of x . The other orthogonal directions, y and z would have gravitational fields of the same strength as g_{11} at the same radius r . Figure 8 is a schematic illustration of that effect. To determine the full metric, it is necessary to label the x direction as r , convert the y and z contributions to r , θ and φ and add them. When that is done, the general expression for the space-time metric becomes

$$ds^2 = g_{44}dt^2 + |g_{11}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)| \quad (28)$$

The g_{11} and r^2 terms are added because they deal with the spread of the gravitational field through all three orthogonal directions. Because g_{11} is negative, equation (28) becomes

$$ds^2 = g_{44}dt^2 - g_{11}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (28a)$$

Fitting Einstein's calculated terms for g_{11} and g_{44} into equation (28a) gives the exact solutions to Einstein's field equations as

$$ds^2 = \frac{dt^2}{\left(1 + \frac{\alpha}{r}\right)} - dr^2\left(1 + \frac{\alpha}{r}\right) - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (29)$$

It is herein designated as the Einstein metric.

Einstein's gravitational work has led to three gravitational metrics. His 1916 paper on 'The Foundations of the General

Theory of Relativity' [14] leads directly to equation (29). When others expanded his work, they included the approximations he used, as shown in table 5. When they are inserted into either of equation (28), they give equation (12), namely

$$ds^2 = dt^2\left(1 - \frac{\alpha}{r}\right) - \frac{dr^2}{\left(1 - \frac{\alpha}{r}\right)} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (12a)$$

The expansion of Einstein's paper [19, 20] presented above, derived equation (11), repeated here for ease of comparison.

$$ds^2 = dt^2e^{-\alpha/r} - dr^2e^{\alpha/r} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (11a)$$

The differences between the three metrics make no difference for situations such as accurate global positioning systems and international timing. It does not make any difference to the magnitude of the anomalous precession in Mercury's perihelion. It does make a change to the direction of that precession. The 'strong field' solution implies orbital regression.

8. Discussion

Table 6 lists some properties of the different metrics. The field status relates to Newton's inverse square law. As far as gravitational attraction is concerned, when $r \gg \alpha$, equations (11), (12) and (29) all trend to Newtonian

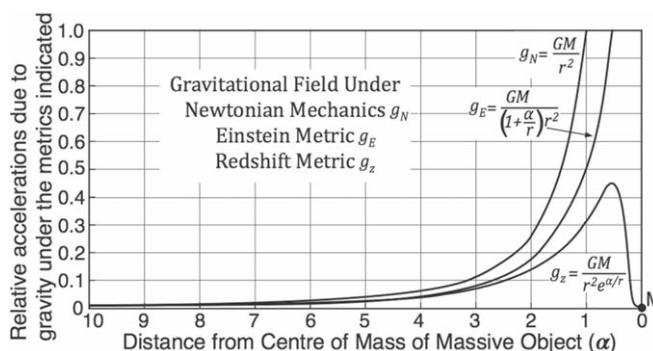


Figure 9. Plots of the relative accelerations due to gravity under Newton's theory, Einstein's metric and this redshift study metric.

Table 6. Some gravitational properties under the different gravitational theories and metrics.

Metric	Gravity	Field Status	Orbit Status	Black Hole
Newton	$\frac{GM}{r^2}$		No Change	No
Einstein	$\frac{GM}{\left(1 + \frac{\alpha}{r}\right)^2}$	Weak	Precess	No
Schwarzschild	$\frac{GM}{\left(1 - \frac{\alpha}{r}\right)^2}$	Strong	Regress	Yes
Redshift	$\frac{GM}{r^2 e^{\alpha/r}}$	Weak	Precess	No

gravity. The g_{11} terms separate the equations (11), (12) and (29) gravitational fields from Newtonian inverse square gravity. Einstein acknowledged his equations were approximations.

Equation (8) shows that, under those conditions, $z = \frac{\alpha}{2r}$. That changes the denominators for the gravitational acceleration equations derived from equations (29), (12) and (11) to $(1 + z)r^2$, $(1 - z)r^2$ and $[(1 + z)r]^2$ respectively. As long as there is no detectable effect of redshift, Newton's inverse square law of gravity will match all observations.

An important feature of table 6 is that the exact solutions to Einstein's gravitational field calculations, equation (29), and the exact extension of his 1911 paper, equation (11), both predict that gravity will be weaker than inverse square law. It follows that those equations predict orbital precession and do not predict the existence of black holes.

The prediction of black holes requires gravity to be stronger than inverse square—'the strong field solution'. When gravity is stronger than inverse square, the orbiting body will travel closer to the attracting body and complete a perihelion-to-perihelion orbit slightly earlier. That would cause its perihelion position to regress.

The accurate prediction of the anomalous precession of Mercury's perihelion and the observation of the precession of star S2 orbiting the massive object in Sagittarius A [40, 41] at the centre of our galaxy, confirm orbital precession has been detected. They require the gravitational attraction to be less than inverse square. Those observations physically exclude the possibility that mass has distorted space-time to make gravity stronger than inverse square, excluding the possibility of black holes.

Figure 9 shows plots of the relative gravitational field strengths due to Einstein's metric, the redshift metric and Newton's inverse square law. The Einstein gravity curve was drawn from calculations of g_E , done in the same manner as those in table 4. It shows the differences between the Einstein and redshift metric gravitational field strengths. The Einstein metric has approximately the same field strength as the redshift metric until about $r = 3\alpha$. After that it rises rapidly.

The g_E gravitational formula is Newton's inverse square law multiplied by $|g_{11}|$. Einstein's $|g_{11}| = \left(1 + \frac{\alpha}{r}\right)$ equates to $g_{11} = (1 + 2z)$. At $z \ll 1$, the valid region of the approximations Einstein used, $(1 + 2z) \approx (1 + z)^2 = e^{\alpha/r}$, which is the redshift metric modification to Newton's inverse square law. That is presented as a further indication that his $g_{\mu\nu}$ terms were the variations with redshift terms associated with Einstein's mass distorting space-time theory of gravity.

At $r \gg \alpha$, it is not possible to distinguish between the Einstein and redshift metrics because $e^{\alpha/2r} \approx 1 + \frac{\alpha}{2r}$. They predict the same values within experimental error.

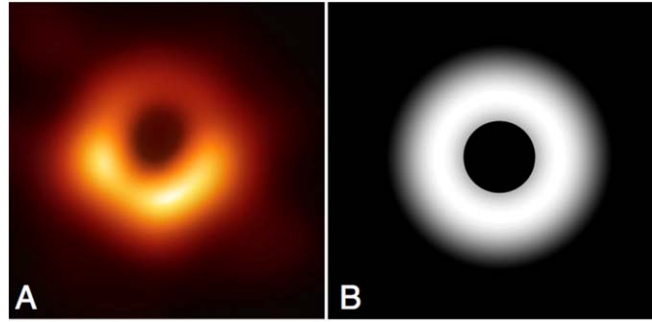


Figure 10. Image from the Event Horizon Telescope collaboration, showing the bright torus shaped structures detected in the centre of galaxy M87 [16, 17], A Reproduced from [17]. CC BY 4.0. B shows the expected torus shape of emissions from a massive object under this redshift metric. B was adjusted to the same image size as A.

They both predict orbital precession. Figure 9 shows the only difference is in the region $r \approx \alpha$. The only results available in that region are from the Event Horizon Telescope collaboration images, one of which is shown in figure 10(A).

It was presented as being of a black hole at the centre of galaxy M87, approximately 55 million light years away. Figure 10(B) is extracted from figure 7 and presented side by side at approximately the same scale. It is suggested their similarity is more than co-incidence. Suggestions were that its mass was about 5 to 7 billion solar masses. It is suggested the mass estimates were based on the Schwarzschild metric.

Predictions under the redshift metric would give a higher mass. There is nothing in the above that excludes mass from accumulating to any amount.

A feature of this work is that the redshift metric is the only one that predicts the torus shape for a massive object and the observed orbital precessions. Those predictions are a good indication that the redshift metric applies for all values of r/α in space–time outside matter. Apart from its inability to predict orbital precession, the Schwarzschild metric is only accurate for $r \gg \alpha$. It has only been tested up to $r \approx 10^{-6}\alpha$. As shown in figure 9, the Einstein metric is good for $r > 3\alpha$.

Figures 5, 6 and 9 and their associated equations, show that when, $r \gg \alpha$ the gravitational field strengths all trend towards Newton's inverse square law values. As all gravitational waves are detected at distances where $r \gg \alpha$ their detection is independent of the metric used for their generation.

Einstein's great contribution was to show that gravity worked by mass distorting space–time. That removed all concern about gravity acting at a distance with no apparent reason. This work shows that space–time distortion was measured by photon redshift. Einstein's work involved determinations of the differences caused by detectable redshift effects.

In the above calculations, that effect was to introduce a redshift term into the denominators in equations (1) and (3). Einstein's field equations approximations changed Newton's r^2 to $r^2\left(1 + \frac{\alpha}{r}\right)$. The doubly approximated Schwarzschild metric changed it to $r^2\left(1 - \frac{\alpha}{r}\right)$. This redshift study changed it to $r^2e^{\alpha/r}$. Beyond those and the extensions that come from them, Newtonian gravity is otherwise unchanged.

It is possible that some authors may have derived the accepted Schwarzschild metric without reference to Einstein's field equations. In that case, they are not solutions to his field equations. They all face the problem that gravity stronger than inverse square physically cannot generate the observed precession of bodies orbiting massive objects that create detectable redshifts.

It is suggested that knowledge of Einstein's equation $ds^2 = g_{\mu\nu}dx_\mu dx_\nu$, his determination of g_{11} , from which g_{44} is automatically derived, plus equations (25) and (28), which include the conversion from Cartesian to Polar coordinates, are sufficient to derive the metrics.

9. Summary and conclusions

Newton's inverse square law of gravity forms the basis of all gravitational studies. Newton gave no reason for gravity to act at a distance. This study started out with Einstein's 1911 approach of applying Newton's gravity to photons that have mass. From that, terms for the variations of photon wavelength and frequency with gravitational field were determined. It showed that a gravitational field increased distances from d to $d(1 + z)$ and decreased frequency from ν to $\nu/(1 + z)$. They are the space and time distortions to which Einstein referred. They were used to match some of his calculations such as the anomalous precession of Mercury's orbit, gravitational lensing and photon redshift. It is believed the calculations were simpler to follow than derivations

used by Einstein and others. Their match shows that it was the same theory. It also shows that Einstein's gravity is founded on distance and time changes associated with photon redshift.

Einstein's mathematical approach and his use of approximations made it difficult for many people to follow his work. By his own admission, Einstein's field equations were approximations. Exact solutions to approximations always remain approximations. This study suggests his approximations were valid for

The Schwarzschild metrics, forwarded as exact solutions to his field equations, were based around incorrect interpretations of the approximations he used. They led to a prediction that gravity could be stronger than inverse square law. That is clearly at odds with observed orbital precessions. This appraisal of Einstein's gravity removed those approximations and derived an exact solution to his field equations. It allowed for orbital precession because gravity was weaker than inverse square. It did not predict the existence of black holes, in which Einstein never believed.

An exact expansion of Einstein's 1911 study was derived. It predicted the strongest gravitational field at $r \approx 0.5\alpha$, if the object's mass was so great that its physical radius was smaller than half its Schwarzschild radius. Particles of relativistic velocity would swirl around near that radius. Their collisions would cause them to emit high energy radiation from a torus shape. Its prediction has the shape and approximate dimensions observed by the Event Horizon Telescope collaboration. That strongly suggests the exact redshift metric, equation (11) is applicable for all values of r/α .

It is believed this redshift metric will match all observations for space-time outside matter. It shows that Einstein's mass distorting space-time is measured by photon redshift. When the effects of redshift are not physically detectable, Newton's inverse square law of gravity will match all observations. It is suggested that vindicates the approximations he used. At $r < 3\alpha$, the redshift metric is required.

The Schwarzschild metrics fail because they are only valid for $r \gg \alpha$ and predict gravity stronger than inverse square. They only allow for the perihelion of orbiting objects to regress against their direction of travel. Appropriate orbiting bodies show a perihelion precession that can only be achieved when gravity is weaker than inverse square. Predictions of black holes have no physical foundation. They also have no foundation in Einstein's gravitational field equations from his general theory of relativity. Einstein never believed in them, suggesting he was right yet again!

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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